6.3 Simple Interest

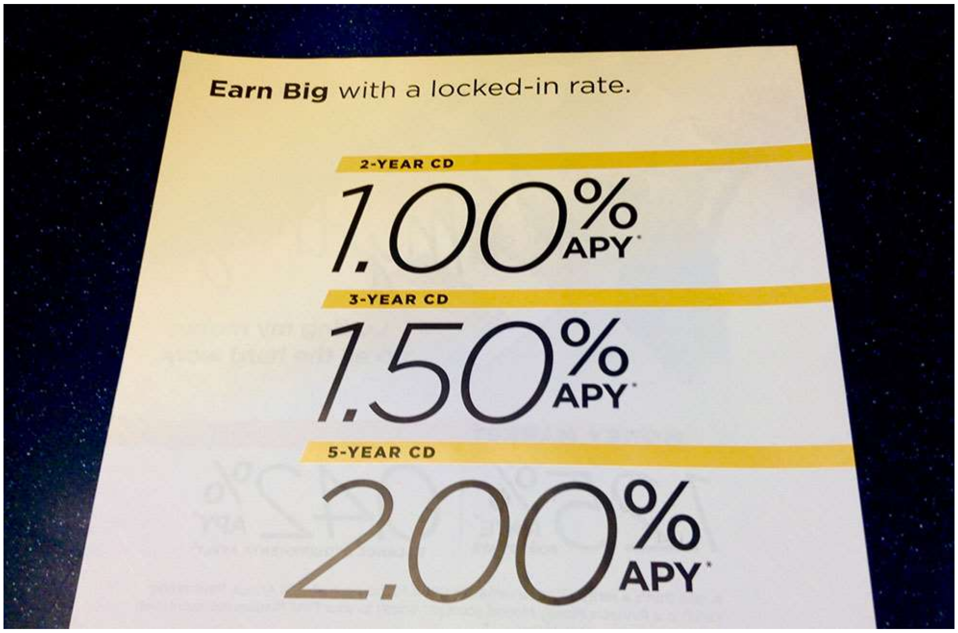

Figure 6.4 Interest is how savings earns money. (credit: "Interest Rates" by Mike Mozart/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Compute simple interest.
2. Understand and compute future value.
3. Compute simple interest loans with partial payments.
4. Understand and compute present value.

There is truth in the phrase "You need to have money to make money." In essence, if you have money to lend, you can lend it at a cost to a borrower and make money on that transaction.

When money is borrowed, the person borrowing the money (borrower) typically has to pay the person or entity that lent the money (the lender) more than the amount of money that was borrowed. This extra money is the **interest** that is to be paid. Interest is sometimes referred to as the cost to borrow, the cost of the loan, or the finance cost.

This idea also applies when someone deposits money in a bank account or some other form of investment. That person is essentially lending the money to the bank or company. The money earned by the depositor is also called interest. The interest is typically based on the amount borrowed, or the **principal**.

The pairing of borrower and lender can take various forms. The borrower may be a consumer using a credit card or taking out a loan from a bank, the lender. Companies also borrow from lending banks. Someone who invests in a company's stock is the lender in this case; the company is essentially the borrower.

In this section, we examine the basic building block of interest paid on loans and borrowed credit and also the returns on investments like bank accounts, simple interest.

Compute Simple Interest

Let's get some terminology understood. Interest to be paid by a borrower is often expressed as an **annual percentage rate**, which is the percent of the principal that is paid as interest for each year the money is borrowed. This means that the more that is borrowed, the more that must be paid back. Sometimes, the interest to be paid back is **simple interest**, which means that the interest is calculated on the amount borrowed only.

The length of time until the loan must be paid off is the **term** of the loan. The date when the loan must be paid off is when the loan is **due**. The day that the loan is issued is the **origination date**. We'll put this terminology to use in the following examples. Note that in this section we will use letters, called variables, to represent the different parts of the formulas we'll be using. This will help keep our formulas and calculations manageable.

Simple Interest Loans with Integer Year Terms

Calculating simple interest is similar to the percent calculations we made in Understanding Percent and Discounts, Markups, and Sales Tax, but must be multiplied by the term of the loan (in years, if dealing with an annual percentage rate).

FORMULA

The simple interest, I, to be paid on a loan with annual interest rate r for a number of years (term of the loan) t, with principal P, is found using $I = P \times r \times t$, where the decimal form of the interest rate, r, is used. The total repaid, then is $T = P + I$ or, more directly, $T = P + P \times r \times t$. This total is often referred to as the loan **payoff amount**, or more simply just the payoff.

When the annual interest rate, the principal, and the number of years that the money is borrowed is known, the interest to be paid can be found and from there the total to be repaid can be calculated.

⚠ *Be aware, interest paid to a lender is almost uniformly rounded up to the next cent.*

EXAMPLE 6.27

Simple Interest on Loans with Integer Year Terms

Calculate the simple interest to be paid on a loan with the given principal, annual percentage rate, and number of years. Then, calculate the loan payoff amount.

1. Principal P = $4,000, annual interest rate r = 5.5%, and number of years t = 4
2. Principal P = $14,800, annual interest rate r = 7.9%, and number of years t = 7

Solution

1. Substitute the principal P = $4,000, the decimal form of the annual interest rate r = 0.055, and number of years t = 4 into the formula for simple interest, and calculate.
   $I = P \times r \times t = 4{,}000 \times 0.055 \times 4 = 880.$
   The simple interest, or cost of the loan, to be paid on the loan is $880.
   The loan payoff amount, or the total to be repaid, is $T = P + I = 4{,}000 + 880 = 4{,}880$, or $4,880.00.
2. Substitute the principal P = $14,800, the decimal form of the annual interest rate r = 0.079, and number of years t = 7 into the formula for simple interest, and calculate.
   $I = P \times r \times t = 14{,}800 \times 0.079 \times 7 = 8{,}184.4.$
   The simple interest, or cost to borrow, to be paid on the loan is $8,184.40.
   The loan payoff amount, or the total to be repaid, is $T = P + I = 14{,}800 + 8{,}184.4 = 22{,}984.4$, or $22,984.40.

YOUR TURN 6.27

Calculate the simple interest to be paid on a loan with the given principal, annual percentage rate, and number of years. Then calculate the loan payoff, or total to be repaid.

1. Principal P = $6,700, annual interest rate r = 11.99%, and number of years t = 3
2. Principal P = $25,800, annual interest rate r = 6.9%, and number of years t = 5

EXAMPLE 6.28

Simple Interest Equipment Loan

Riley runs an auto repair shop, and needs to purchase a new brake lathe, which costs $11,995. She takes out a two-year, simple interest loan at an annual interest rate of 14.9%. How much interest will she pay and how much total will she repay on the loan?

Solution

Step 1. Determine the variables, or parts of the formula. The principal P is the cost of the brake lathe, so P = $11,995.

The interest rate Riley pays is 14.9%, or r = 0.149 in decimal form. The length of the loan is two years, so t = 2. We are first asked to find I, the interest Riley will pay.

Step 2. Substitute the known variables into the formula for simple interest $I = P \times r \times t$ and solve for I.

From Step 1 we have $I = P \times r \times t = 11995 \times 0.149 \times 2 = 3{,}574.51$.

This tells us that the simple interest, or cost to borrow, to be paid on the loan is $3,574.51.

Step 3. Use the formula $T = P + I$ to determine the total amount Riley will repay, T.

The total to be repaid is $T = P + I = 11{,}995 + 3{,}574.51 = 15{,}569.51$, or $15,569.51.

YOUR TURN 6.28

1. Beth is the owner of a small retail store in downtown St. Louis. The windows in the storefront need replacing, so she needs to take out a $9,500 loan to get the repairs done. The rate she secures is 9.25% and the term of the loan is one year. How much interest will she pay and how much total will she repay on the loan?

Simple Interest Loans with Other Lengths of Terms

In the previous example and Your Turn exercise, the loans were paid back in one payment after an integer number of years. However, there are also loans lasting a length of time not equal to an integer number of years (like 1, 2, or 3 years or more), but in a number of months (like 4 months, 18 months, and so on). What model would apply to these situations?

When the loan is paid back after a term that is not an integer number of years but is instead a number of months, the term of the loan, or time, t, is expressed as a fraction of the year. So for a 2-month loan, the time, in years, is 2/12 = 1/6. For a 5-month loan, the time in years is 5/12. For an 18-month term, the term in years is 18/12 = 1.5.

EXAMPLE 6.29

Loan to Purchase Equipment

Abeje needs a loan to purchase equipment for the gym she is going to open. She visits the bank and secures a 4-month loan of $20,000. Her annual percentage rate is 6.75%. How much interest will Abeje pay and what is her loan payoff amount?

Solution

Abeje's loan is for $20,000, so her principal is P = 20,000. The interest rate Abeje will pay is 6.75%, or r = 0.0675 in decimal form. The length of the loan is 4 months, so $t = \frac{4}{12}$. Substituting these in the formula for simple interest, we find her interest to be $I = P \times r \times t = 20{,}000 \times 0.0675 \times 4/12 = 450$

The simple interest, or cost to borrow, to be paid on the loan is $450.00.

The payoff is $T = P + I = 20{,}000 + 450 = 20{,}450$, or $20,450.00.

YOUR TURN 6.29

1. Samuel needs to borrow $8,400 to pay for repairs to his small manufacturing facility. He manages to get a simple interest loan at 17.33%, to be paid after 6 months. How much interest will Samuel pay and what is Samuel's loan payoff amount?

Those examples dealt in months. However, some loans are for days only (45 days, 60 days, 120 days). In such cases, we find the daily interest rate. The fraction we will use for the daily interest rate is the interest rate (as a decimal) divided by 365. This may be referred to as Actual/365. In order to find the term of the loan, divide the number of days in the term of the loan by 365.

FORMULA

To determine the interest, I, on a loan with term t expressed in days, with principal of P, and interest rate in decimal form of r, calculate $I = P \times \frac{r}{365} \times t$. Here, $\frac{r}{365}$ represents the daily interest rate.

Alternately, the above formula is equivalent to $I = P \times r \times \frac{t}{365}$, where the interest rate remains an annual rate, but the time is expressed as a fraction of the year.

WHO KNEW?

It seems reasonable to use 365 as the number of days in the year, since there are 365 days in most years. However, sometimes, banks have used (and continue to use) 360 as the number of days in a year. They may also treat all months as if they have 30 days. These differences lead to (sometimes small) differences in how much interest is paid. Since the number of days is in the denominator, a smaller denominator (360) will result in larger numbers (interest) that is 365 is used for the denominator. See this page from ACRE (https://openstax.org/r/ACRE) for a comparison.

EXAMPLE 6.30

Loan for Moving Costs

David plans to move his family from Raleigh, North Carolina to Tempe, Arizona. His company will reimburse (pay after the move) David for the move. David does research and determines that movers will cost $5,600 to move his family's belongings to Tempe. He takes out a simple interest, 45-day loan at 11.75% interest to pay this cost. How much interest will be paid on this 45-day loan, and what is David's loan payoff amount?

Solution

This loan is in terms of days, so we will use the formula $I = P \times \frac{r}{365} \times t$, where t is the number of days and r is the annual interest rate.

The principal for the loan is the moving cost, or P = 5,600. The annual interest rate that David will pay is 11.75%, which in decimal is 0.1175. The length of time for the loan is 45 days, so t = 45.

Substituting these values into the formula and calculating, we find that the interest to be paid is $I = 5{,}600 \times \frac{0.1175}{365} \times 45 = 81.13$, or $81.13 (remember, interest is almost always rounded up to the next cent).

The payoff for the loan is $5,681.13.

YOUR TURN 6.30

1. Heather runs a silk screen t-shirt shop, and seeks a short-term loan to pay for new inventory (paints, blades, shirts). They secure a $3,700 simple interest loan for 60 days, at an annual rate of 18.99%. How much will borrowing the money cost Heather, and what is her loan payoff amount?

Understand and Compute Future Value

Money can be invested for a specific amount of time and earn simple interest while invested. The terminology and calculations are the same as we've already seen. However, instead of the total to be paid back, the investor is interested in the total value of the investment after the interest is added. This is called the **future value** of the investment.

FORMULA

The future value, FV, of an investment that yields simple interest is $FV = P + I = P + P \times r \times t$, where P is the principal (amount invested at the start), r is the annual interest rate in decimal form, and t is the length of time the money is invested. The time t will be an integer if the term of the deposit is an integer number of years, will be number of months/12 if the term is in months, will be actual/365 if the deposit is for a number of days.

EXAMPLE 6.31

Simple Interest on a Deposit

In the following, determine how much interest was earned on the investment and the future value of the investment, if the investment yields simple interest.

1. Principal is $1,000, annual interest rate is 2.01%, and time is 5 years
2. Principal is $5,000, annual interest rate is 1.85%, and time is 30 years
3. Principal is $10,000, annual interest rate is 1.25%, and time is 18 months
4. Principal is $7,000, annual interest rate is 3.26%, and time is 100 days

Solution

1. The principal is P = $1,000, the annual interest rate, in decimal form, is 0.0201, and the term is 5 years, or t = 5. Since the term is an integer number of years, the interest earned on the investment is $I = P \times r \times t = 1{,}000 \times 0.0201 \times 5 = 100.5$, or the interest earned was $100.50.
   To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 1{,}000 + 100.5 = 1100.5$. The future value of the investment at the end of 5 years is $1,100.50.
   Notice that the future value could have been calculated directly with $FV = P + P \times r \times t$
2. The principal is P = $5,000, the annual interest rate, in decimal form, is 0.0185, and the term is 30 years, or t = 30. Since the term is an integer number of years, the interest earned on the investment is $I = P \times r \times t = 5{,}000 \times 0.0185 \times 30 = 2{,}775$, or the interest earned was $2,775.00. To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 5{,}000 + 2{,}775 = 7{,}775$. The future value of the investment at the end of 30 years is $7,775.00.
3. The principal is P = $10,000, the annual interest rate, in decimal form, is 0.0125, and the term is 18 months. Since the term is in months, we have to write the months in terms of years. For 18 months, we use 18/12 as t. The interest earned on the investment is $I = P \times r \times t = 10{,}000 \times 0.0125 \times \frac{18}{12} = 187.5$, or the interest earned was $187.50. To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 10{,}000 + 187.5 = 10{,}187.5$. The future value of the investment at the end of 18 months is $10,187.50.
4. Principal is $7,000, annual interest rate is 3.26%, and time is 100 days. The principal is P = $7,000, the annual interest rate, in decimal form, is 0.0326, and the term is 100 days. Since the term is in days, we have to write the time using actual/365, or t = 100/365. The interest earned on the investment is $I = P \times r \times t = 7{,}000 \times 0.0326 \times \frac{100}{365} = 62.52$, or the interest earned was $62.52. To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 7{,}000 + 62.52 = 7{,}062.52$. The future value of the investment at the end of 100 days is $7,062.52.

YOUR TURN 6.31

In the following, determine how much interest was earned on the investment and the future value of the investment if the investment yields simple interest.

1. Principal is $4,500, annual interest rate is 1.88%, and time is 3 years
2. Principal is $2,000, annual interest rate is 2.03%, and time is 10 years
3. Principal is $120,000, annual interest rate is 3.1%, and time is 100 days
4. Principal is $4,680, annual interest rate is 1.55%, and time is 42 months

You may have noticed that for these problems, the future value was rounded down. When the future value is paid, the amount is typically rounded down.

A certificate of deposit (CD) is a savings account that holds a single deposit (the principal) for a fixed term at a fixed interest rate. Once the term of the CD is over, the CD may be redeemed (cashed in or withdrawn) and the owner of the

CD receives the original principal plus the interest earned. The deposit often cannot be withdrawn until the term is up; if it can be withdrawn early, there is often a penalty imposed to do so.

EXAMPLE 6.32

Certificate of Deposit

Jonas deposits $2,500 in a CD bearing 3.25% simple interest for a term of 3 years. When he redeems his CD at the end of the 3 years, how much will he receive?

Solution

This is a future value example. We know that P = $2,500 is the amount deposited. The annual simple interest rate in decimal form is r = 0.0325. The term of the investment is t = 3 years.

Substituting those values into the future value formula, we have

$$FV = P + P \times r \times t = 2{,}500 + 2{,}500 \times 0.0325 \times 3 = 2{,}500 + 243.75 = 2{,}743.75.$$

When the CD is redeemed, Jonas will receive $2,743.75.

YOUR TURN 6.32

1. Mia deposits $4,900 in a CD bearing 3.95%. The CD term is 7 years. When she redeems the CD, how much will Mia receive?

WORK IT OUT

The reason CD (certificate of deposit) rates look so small is because they are extremely safe investments. Though overall interest rates for CDs change over time and individual returns vary with the terms of the CD, investors are offered predictable interest income for their investments.

To investigate this yourself, search online to determine the strengths and weaknesses of CDs (investopedia.com offers good, basic information on investing). Then, online, identify five national banks and two local banks who offer CDs.

- Track the interest rates for the CDs at various terms (1 year, 3 years, 5 years) for each of the banks you found that offer CDs.
- Calculate the amount of interest earned for a $10,000 deposit for each CD at each of the terms.
- Compare the results from the various banks, CDs, and terms and decide which is the best investment. You may want to consider both the length of time that the money is locked up, and the return.

Paying Simple Interest Loans with Partial Payments

In every example above, there was one payment for the loan, or one withdrawal for the investment. However, for many loans (house, car, in-ground swimming pool), the loan will be paid back in two or more payments. Such a payment is called a **partial payment**, because they only pay off part of the loan.

When a partial payment is made, some of the payment pays for the principal, but the rest of the payment pays for interest on the principal. When making the first partial payment, the interest is calculated on the principal for the time between the origination date of the loan and the date of the payment. If another partial payment is made, the interest is calculated based on the remaining principal and the time between the previous partial payment and the current partial payment date.

EXAMPLE 6.33

Interest Paid in a Partial Payment on a Loan

1. A simple interest loan for $6,500 is taken out at 12.6% annual percentage rate. A partial payment is made 45 days into the loan period. How much of the partial payment will be for interest?

2. A simple interest loan for $13,700 is taken out at 6.55% annual interest rate. A partial payment is to be made after 60 days. How much of the partial payment will be for interest?

Solution

1. To find the interest paid in this partial payment, we calculate the interest on the principal for the time between the origination of the loan and the payment day, or 45 days.
   The principal is $6,500. The annual interest rate, in decimal form, is 0.126.
   The interest paid for 45 days is found by substituting the values for principal P, rate r, and time t into the formula $I = P \times \frac{r}{365} \times t$.
   Calculating, we have $I = P \times \frac{r}{365} \times t = 6{,}500 \times \frac{0.126}{365} \times 45 = 100.9726$. Rounding up, the portion of the partial payment that will be paid for interest is $100.98.
2. To find the interest paid in this partial payment, we calculate the interest on the principal for the time between the origination of the loan and the payment day, or 60 days.
   The principal is $13,700. The annual interest rate, in decimal form, is 0.0655.
   The interest paid for those 60 days is found by substituting those values into the formula $I = P \times \frac{r}{365} \times t$.
   Calculating, we have $I = P \times \frac{r}{365} \times t = 13{,}700 \times \frac{0.0655}{365} \times 60 = 147.5096$. Rounding up, the portion of the partial payment that will be paid for interest is $147.51.

YOUR TURN 6.33

1. A simple interest loan for $50,000 is taken out at 5.15% annual percentage rate. A partial payment is made 120 days into the loan period. How much of the partial payment will be for interest?
2. A simple interest loan for $8,500 is taken out at 9.9% annual interest rate. A partial payment is to be made after 75 days. How much of the partial payment will be for interest?

Remaining Balance

The previous examples demonstrated how to determine the interest paid in a partial payment. Using this, we can determine the remaining balance after a partial payment.

Step 1: determine the amount of the payment, P, that is applied to interest, I.

Step 2: subtract the amount paid in interest from the payment, $(P - I)$. This is the amount applied to the balance.

Step 3: subtract the amount applied to the balance (the value obtained in Step 2) from the balance of the loan, $B - (P - I)$. This is the remaining balance after the partial payment.

EXAMPLE 6.34

Determining the Remaining Balance on a Loan After a Partial Payment

1. A simple interest loan for $45,500 is taken out at 11.8% annual percentage rate. A partial payment of $20,000 is made 50 days into the loan period. After this payment, what will the remaining balance of the loan be?
2. A simple interest loan for $150,000 is taken out at 5.85% annual percentage rate. A partial payment of $50,000 is made 70 days into the loan period. After this payment, what will the remaining balance of the loan be?

Solution

1. The principal is $45,500, which will be treated as the balance, B, of the loan. The annual simple interest rate, in decimal form, is 0.118. The time is t = 50 days.
   Step 1: Determine the amount of the partial payment that is applied to interest. To find this, substitute the values above into the formula $I = P \times \frac{r}{365} \times t$ and calculate. Calculating, the amount of the payment that is applied to interest is $I = 45{,}500 \times \frac{0.118}{365} \times 50 = 735.4795$. Rounding up, we have I = $735.48.
   Step 2: The amount of the payment that is to be applied to the balance of the loan is partial payment minus the amount of the partial payment that is applied to the interest. The payment is $2,000. The amount that is applied to the balance is $P - I = \$20{,}000 - \$735.48 = \$19{,}264.52$.
   Step 3: The remaining balance is found by subtracting the amount applied to the balance from the previous balance, or $B - (P - I) = \$45{,}500 - \$19{,}264.52 = \$26{,}235.48$.

The remining balance after the partial payment is $26,235.48.

2. The principal is $150,000, which will be treated as the balance, B, of the loan. The annual simple interest rate, in decimal form, is 0.0585. The time is t = 70 days.
   Step 1: Determine the amount of the partial payment that is applied to interest. To find this, substitute the values above into the formula $I = P \times \frac{r}{365} \times t$ and calculate. Calculating, the amount of the payment that is applied to interest is $I = 150{,}000 \times \frac{0.0585}{365} \times 70 = 1{,}682.8767$. Rounding up, we have I = $1,682.88.
   Step 2: The amount of the payment that is to be applied to the balance of the loan is partial payment minus the amount of the partial payment that is applied to the interest. The payment is $50,000. The amount that is applied to the balance is $P - I = \$50{,}000 - \$1{,}682.88 = \$48{,}317.12$.
   Step 3: The remaining balance is found by subtracting the amount applied to the balance from the previous balance, or $B - (P - I) = \$150{,}000 - \$48{,}317.12 = \$101{,}682.88$.
   The remining balance after the partial payment is $101,682.88.

YOUR TURN 6.34

1. A simple interest loan for $1,400 is taken out at 12.5% annual percentage rate. A partial payment of $700 is made 20 days into the loan period. After this payment, what will the remaining balance of the loan be?
2. A simple interest loan for $23,000 is taken out at 7.25% annual percentage rate. A partial payment of $10,000 is made 40 days into the loan period. After this payment, what will the remaining balance of the loan be?

Loan Payoff

Finally, we will determine the amount to be paid at the end of the loan. To do so, we apply the formula for the loan payoff to the remaining balance. However, the length of time for that remaining balance is the time between the partial payment and the day the loan is paid off.

Step 1: Determine the remaining balance after the partial payment.

Step 2: Calculate the number of days between the partial payment and the date the loan is paid off. This will be the time t in the payment formula.

Step 3: Calculate the amount to be paid at the end of the loan, or the payoff amount, using $\text{Payoff} = P + P \times \frac{r}{365} \times t$, where P is the remaining balance and t is the time found in Step 2.

EXAMPLE 6.35

Finding Loan Pay Off After a Partial Payment

Laura takes out an $18,400 loan for 120 days at 17.9% simple interest. She makes a partial payment of $7,500 after 45 days. What is her payoff amount at the end of the loan?

Solution

The initial balance, or principal, of her loan is $18,400. The interest rate in decimal form is 0.179. Her partial payment of $7,500 is made after 45 days. Using these values, we can determine how much of the partial payment is applied to the balance. From there, we can determine her final loan payoff after 120 days.

Step 1: Determine the remaining balance after the partial payment. Using the partial payment process outlined in the previous example, we first find that the amount of the partial payment that is applied to the balance. Their interest paid in the partial payment is $I = P \times \frac{r}{365} \times t = 18{,}400 \times \frac{0.179}{365} \times 45 = 406.0603$, or $406.07 (remember to round up!). Using this and that the loan amount was for $18,400, the remaining balance on the loan after the partial payment is $B - (P - I) = \$18{,}400 - (\$7{,}500 - \$406.07) = \$11{,}306.07$.

Step 2: The number of days between the partial payment and the date that the loan is to be paid off is 120 – 45 = 75. This means that the time between the partial payment and the final payment is 75 days.

Step 3: To calculate the payoff amount, use $\text{payoff} = P + P \times \frac{r}{365} \times t$, with P = $11,306.07 (the remaining balance), t = 75 (from Step 2) and r = 0.179. The payoff amount, then, is $\text{payoff} = 11{,}306.07 + 11{,}306.07 \times \frac{0.179}{365} \times 75 = 11{,}721.9165$. Rounding up, the payoff amount is $11,721.92.

YOUR TURN 6.35

1. Paola takes out a 75-day loan for $3,500.00. Her interest rate is 11.2%. If she makes a partial payment of $1,250.00 after 30 days, what will her payoff be at the end of the loan?

Repeated Partial Payments

Car loans and mortgages (loans for homes) are paid off through repeated partial payments, most often monthly payments. Since car loans are often 3 to 6 years, and mortgages 15 to 30 years, calculating each individual monthly payment one at a time is time consuming and tedious. Even a 3-year loan would involve applying the above steps 36 times! Fortunately, there is a formula for determining the amount of each partial payment for monthly payments on a simple interest loan.

FORMULA

The amount of monthly payments, A, for a loan with principal P, monthly simple interest rate r (in decimal form), for t number of months is found using the formula $A = P \times \frac{r \times (1 + r)^t}{(1 + r)^t - 1}$. The monthly interest rate is the annual rate divided by 12. The number of months is the number of years times 12.

EXAMPLE 6.36

Calculating Car Payments

Desiree buys a new car, by taking a loan out from her credit union. The balance of her loan is $27,845.00. The annual interest rate that Desiree will pay is 7.3%. She plans to pay this off over 4 years. How much will Desiree's monthly payment be?

Solution

To use the formula for monthly payments, we need the principal, the interest rate, and the number of years. The principal is $27,845. The annual rate, in decimal form, is 0.073. Dividing 0.073 by 12 gives the monthly interest rate $0.073/12 = 0.00608\overline{3}$. She takes the loan out for 4 years, which is $t = 12 \times 4 = 48$ months. Substituting these values into the formula, $A = P \times \frac{r \times (1 + r)^t}{(1 + r)^t - 1}$, we calculate:

$$\begin{aligned} A &= 27{,}845 \times \frac{0.00608\overline{3} \times (1 + 0.00608\overline{3})^{48}}{(1 + 0.00608\overline{3})^{48} - 1} \\ &= 27{,}845 \times \frac{0.00608\overline{3} \times (1.00608\overline{3})^{48}}{(1.00608\overline{3})^{48} - 1} \\ &= 27{,}845 \times \frac{0.00608\overline{3} \times 1.337918996}{1.337918996 - 1} \\ &= 27{,}845 \times \frac{0.008139007}{0.337918996} \\ &= 27{,}845 \times 0.024085675 \\ &= 670.6656 \end{aligned}$$

Using the formula and rounding up to the next cent, we see that Desiree's monthly payment will be $670.67.

YOUR TURN 6.36

1. Russell buys a new car, by taking a loan out from the dealership. After all of the discussions are over, he finances (gets a loan for) $23,660. The annual interest rate that Russell will pay is 4.76%. He plans to pay this off over 5 years. How much will Russell's monthly payment be?

TECH CHECK

The calculation of payments is long, and involves many steps. However, most spreadsheet programs, including Google Sheets, have a payment function. In Google Sheets, that function is PMT. To find the payment for an installment loan (like for a car), you need to enter the interest rate per period, the number of payments, and the loan amount. From Your Turn 6.36, the rate was 0.0476/12, the number of payments was 60, and the loan amount was $23,660. In Google Sheets, select any cell and enter the following:

=PMT(0.0476/12,60,23660)

And click the enter key. Immediately, in the cell you selected, the payment of $443.90 appears, though with a negative sign. The negative sign indicates it is a payment out of an account. Since we want to know the payment amount, we ignore the negative sign. The result, with the formula in the formula bar, is shown in Figure 6.5.

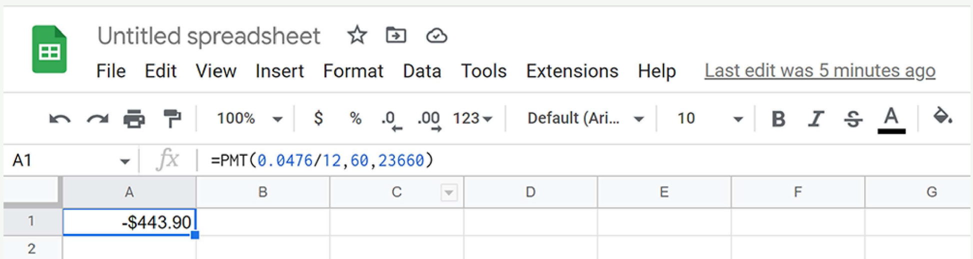

Figure 6.5 Google Sheets payment function

In general, to use the PMT function in Google Sheets, enter

=PMT(r/12,t*12,P)

where r is the annual interest rate, t is the number of years, and P is the principal of the loan.

Understand and Compute Present Value for Simple Interest Investments

When finding the future value of an investment, we know how much is deposited, but we have no idea how much that money will be worth in the future. If we set a goal for the future, it would be useful to know how much to deposit now so an account reaches the goal. The amount that needs to be deposited now to hit a goal in the future is called the **present value**.

FORMULA

The present value, PV, of money deposited at an annual, simple interest rate of r (in decimal form) for time t (in years) with a specified future value of FV, is calculated with the formula $PV = \frac{FV}{(1 + rt)}$.

Note: Present value, in this calculation, is always rounded up. Otherwise, future value may fall short of the target future value.

Understanding what this tells you is important. When you find the present value, that is how much you need to invest now to reach the goal FV, under the conditions (time and rate) at which the money will be invested.

EXAMPLE 6.37

Compute the present value of the investment described. Interpret the result.

1. FV = $10,000, t = 15 years, annual simple interest rate of 5.5%
2. FV = $150,000, t = 20 years, annual simple interest rate of 6.25%
3. FV = $250,000, t = 486 months, annual simple interest rate of 4.75%

Solution

1. The future value is FV = $10,000. The time of the investment is in years, so t = 15. The annual, simple interest rate is 5.5%, which in decimal form is 0.055. We substitute those values into the formula and calculate. $PV = \frac{FV}{(1+rt)} = \frac{10{,}000}{(1+(0.055)\times(15))} = \frac{10{,}000}{(1+0.825)} = \frac{10{,}000}{(1.825)} = 5{,}479.452$. Rounding up, we see that the present value of $10,000 invested at a simple annual interest rate of 5.5% for 15 years is $5,479.46. This means that $5,479.46 needs to be invested so that, after 15 years at 5.5% interest, the investment will be worth $10,000.
2. The future value is FV = $150,000. The time of the investment is in years, so t = 20. The annual, simple interest rate is 6.25%, which in decimal form is 0.0625. We substitute those values into the formula and calculate. $PV = \frac{FV}{(1+rt)} = \frac{150{,}000}{(1+(0.0625)\times(20))} = \frac{150{,}000}{(1+1.25)} = \frac{150{,}000}{(2.25)} = 66{,}666.\overline{6}$. Rounding up, we see that the present value of $150,000 invested at a simple annual interest rate of 6.25% for 20 years is $66,666.67. This means that $66,666.67 needs to be invested so that, after 20 years at 6.25% interest, the investment will be worth $150,000.
3. The future value is FV = $250,000. The time of the investment is 486 months. This needs to be converted to years. To do so, divide the number of months by 12, giving $years = \frac{486}{12} = 40.5$, so t = 40.5 years. The annual, simple interest rate is 4.75%, which in decimal form is 0.0475. We substitute those values into the formula and calculate. $PV = \frac{FV}{(1+rt)} = \frac{250{,}000}{(1+(0.0475)\times(40.5))} = \frac{150{,}000}{(1+1.92375)} = \frac{150{,}000}{(2.92375)} = 129{,}954.5159$. Rounding up, we see that the present value of $250,000 invested at a simple annual interest rate of 4.75% for 486 months is $129,954.52. This means that $129,954.52 needs to be invested so that, after 486 months at 4.75% interest, the investment will be worth $150,000.

YOUR TURN 6.37

Compute the present value of the investment described. Interpret the result.

1. FV = $25,000, t = 10 years, annual simple interest rate of 7.5%
2. FV = $320,000, t = 35 years, annual simple interest rate of 6.5%
3. FV = $90,000, t = 270 months, annual simple interest rate of 3.75%

EXAMPLE 6.38

Present Value of a CD

Beatriz will invest some money in a CD that yields 3.99% simple interest when invested for 30 years. How much must Beatriz invest so that after those 30 years, her CD is worth $300,000?

Solution

Beatriz needs to know how much to deposit now so that her CD is worth $300,000 after 30 years. This means she needs to know the present value of that $300,000. The time is 30 years and the annual simple interest rate, in decimal form, is 0.0399. Using that information and the formula for present value, we calculate the present value of that $300,000. $PV = \frac{FV}{(1+rt)} = \frac{300{,}000}{(1+(0.0399)\times(30))} = \frac{300{,}000}{(1+1.197)} = \frac{300{,}000}{(2.197)} = 136{,}549.8407$. Rounding up, Beatriz needs to invest $136,549.85 so that she has $300,000 in 30 years.

YOUR TURN 6.38

1. Kentaro will invest some money in a CD that yields 2.75% simple interest when invested for 5 years. How much must Kentaro invest so that after those 5 years his CD is worth $12,000?

Check Your Understanding

13. What is interest?

14. What is the principal in a loan?

15. Calculate the simple interest to be paid for a 6-year loan with principal $1,500.00 and annual interest rate of 12.99%

16. A simple interest loan for $24,200 is taken out at 10.55% annual percentage rate. A partial payment of $13,000 is made 25 days into the loan period. After this payment, what will the remaining balance of the loan be?
17. Find the monthly payment for a $9,800.00 loan at a 13.8% interest for 4 years.
18. Find the present value of an investment with future value $30,000 with a simple interest rate of 3.75% invested for 10 years.

SECTION 6.3 EXERCISES

1. If $1,500.00 is invested in an account bearing 3.5% interest, what is the principal?
2. If $1,500 is invested in an account bearing 3.5% interest, what is the interest rate?
3. What is simple interest?
4. What is the present value of an investment?
5. What is the future value of an investment?
6. What is a partial payment on a loan?

In the following exercises, calculate the simple interest and payoff for the loan with the given principal, simple interest rate, and time.

7. Principal P = $5,000, annual interest rate r = 6.5%, and number of years t = 6
8. Principal P = $3,500, annual interest rate r = 12%, and number of years t = 7
9. Principal P = $7,800, annual interest rate r = 11.5%, and number of years t = 10
10. Principal P = $62,500, annual interest rate r = 4.88%, and number of years t = 4
11. Principal P = $4,600, annual interest rate r = 9.9%, for 18 months
12. Principal P = $19,000, annual interest rate r = 16.9%, for 14 months
13. Principal P = $8,500, annual interest rate r = 10.66%, for 6 months
14. Principal P = $17,600, annual interest rate r = 17.9%, for 20 months
15. Principal P = $4,000, annual interest rate r = 8.5%, for 130 days
16. Principal P = $9,900, annual interest rate r = 15.9%, for 90 days
17. Principal P = $600, annual interest rate r = 16.8%, for 25 days
18. Principal P = $890, annual interest rate r = 9.75%, for 200 days

In the following exercises, find the future value of the investment with the given principal, simple interest rate, and time.

19. Principal is $5,300, annual interest rate is 2.07%, and time is 18 years.
20. Principal is $14,700, annual interest rate is 3.11%, and time is 10 years.
21. Principal is $5,600, annual interest rate is 2.55%, for 30 months.
22. Principal is $10,000, annual interest rate is 1.99%, for 15 months.
23. Principal is $2,000, annual interest rate is 3.22%, for 100 days.
24. Principal is $900, annual interest rate is 3.75%, for 175 days.

In the following exercises, determine the amount applied to principal for the indicated partial payment on the loan with the given principal, interest rate, and time when the partial payment was made.

25. A simple interest loan for $2,700 is taken out at 11.6% annual percentage rate. A partial payment of $1,500 is made 28 days into the loan period.
26. A simple interest loan for $900 is taken out at 18.9% annual percentage rate. A partial payment of $400 is made 30 days into the loan period.
27. A simple interest loan for $13,500 is taken out at 14.8% annual percentage rate. A partial payment of $8,000 is made 75 days into the loan period.
28. A simple interest loan for $9,900 is taken out at 9.875% annual percentage rate. A partial payment of $4,000 is made 65 days into the loan period.

In the following exercises, determine the remaining principal for the indicated partial payment on the loan with the given principal, interest rate, and time when the partial payment was made.

29. A simple interest loan for $2,700 is taken out at 11.6% annual percentage rate. A partial payment of $1,500 is made 28 days into the loan period.
30. A simple interest loan for $900 is taken out at 18.9% annual percentage rate. A partial payment of $400 is made 30 days into the loan period

31. A simple interest loan for $13,500 is taken out at 14.8% annual percentage rate. A partial payment of $8,000 is made 75 days into the loan period.
32. A simple interest loan for $9,900 is taken out at 9.875% annual percentage rate. A partial payment of $4,000 is made 65 days into the loan period.

In the following exercises, find the payoff value of the loan with the given principal, annual simple interest rate, term, partial payment, and time at which the partial payment was made.

33. Principal = $1,500, rate = 6.99%, term is 5 years, partial payment of $900 made 2 years into the loan.
34. Principal = $21,500, rate = 7.44%, term is 10 years, partial payment of 15,000 made after 6 years.
35. Principal = $6,800, rate = 11.9%, term is 200 days, partial payment of $4,000 made after 100 days.
36. Principal = $800, rate = 13.99%, term is 150 days, partial payment of $525 made after 50 days.

In the following exercises, find the monthly payment for a loan with the given principal, annual simple interest rate and number of years.

37. Principal = $4,500, rate = 8.75%, years = 3
38. Principal = $2,700, rate = 15.9%, years = 5
39. Principal = $13,980, rate = 10.5%, years = 4
40. Principal = $8,750, rate = 9.9%, years = 10

In the following exercises, find the present value for the given future value, FV, annual simple interest rate r, and number of years t.

41. FV = $25,000, t = 15 years, annual simple interest rate of 6.5%
42. FV = $12,000, t = 10 years, annual simple interest rate of 4.5%
43. FV = $15,000, t = 16 years, annual simple interest rate of 3.5%
44. FV = $100,000, t = 30 years, annual simple interest rate of 5.5%

45. Rita takes out a simple interest loan for $4,000 for 5 years. Her interest rate is 7.88%. How much will Rita pay when the loan is due?

46. Humberto runs a private computer networking company, and needs a loan of $31,500 for new equipment. He shops around for the lowest interest rate he can find. He finds a rate of 8.9% interest for a 10-year term. How much will Humberto's payoff be at the end of the 10 years?

47. Jaye needs a short-term loan of $3,500. They find a 75-day loan that charges 14.9% interest. What is Jaye's payoff?

48. Theethat's car needs new struts, which cost $1,189.50 installed, but he doesn't have the money to do so. He asks the repair shop if they offer any sort of financing. It offers him a short-term loan at 18.9% interest for 60 days. What is Theethat's payoff for the struts?

49. Michelle opens a gaming shop in her small town. She takes out an $8,500 loan to get started. The loan is at 9.5% interest and has a term of 5 years. Michelle decides to make a partial payment of $4,700 after 3 years. What will Michelle pay when the loan is due?

50. A small retailer borrows $3,750 for a repair. The loan has a term of 100 days at 13.55% interest. If the retailer pays a partial payment of $2,000 after 30 days, what will the loan payoff be when the loan is due?

51. Sharon invests $2,500 in a CD for her granddaughter. The CD has a term of 5 years and has a simple interest rate of 3.11%. After that 5-year period, how much will the CD be worth?

52. Jen and Fred have a baby, and deposit $1,500 in a savings account bearing 1.76% simple interest. How much will the account be worth in 18 years?

53. Yasmin decides to buy a used car. Her credit union offers 7.9% interest for 5-year loans on used cars. The cost of the car, including taxes and fees, is $11,209.50. How much will Yasmin's monthly payment be?

54. Cleo runs her own silk-screening company. She needs new silk-screening printing machines, and finds two that will cost her, in total, $5,489.00. She takes out a 3-year loan at 8.9% interest. What will her monthly payments be for the loan?

55. Kylie wants to invest some money in an account that yields 4.66% simple interest. Her goal is to have $20,000 in 15 years. How much should Kylie invest to reach that goal?

56. Ishraq wants to deposit money in an account that yields 3.5% simple interest for 10 years, to help with a down payment for a home. Her goal is to have $25,000 for the down payment. How much does Ishraq need to deposit to reach that goal?

In the following exercises, use the **Cost of Financing**. The difference between the total paid for a loan, along with all other charges paid to obtain the loan, and the original principal of the loan is the cost of financing. It measures how much more you paid for an item than the original price. In order to find the cost of financing, find the total paid over the life of the loan. Add to that any fees paid for the loan. Then subtract the principal.

$\text{Cost of Financing} = \text{Total of payments} + \text{fees} - \text{principal}.$

57. Yasmin decides to buy a used car. Her credit union offers 7.9% interest for 5-year loans on used cars. The cost of the car, including taxes and fees, is $11,209.50. How much did she pay the credit union over the 5 years? What was the cost of financing for Yasmin?

58. Cleo runs her own silk-screening company. She needs new silk-screening printing machines, and finds two that will cost her, in total, $5,489.00. She takes out a 3-year loan at 8.9% interest. What was the cost of financing for Cleo?

6.4 Compound Interest

Figure 6.6 The impact of compound interest (credit: "English Money" by Images Money/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Compute compound interest.
2. Determine the difference in interest between simple and compound calculations.
3. Understand and compute future value.
4. Compute present value.
5. Compute and interpret effective annual yield.

For a very long time in certain parts of the world, interest was not charged due to religious dictates. Once this restriction was relaxed, loans that earned interest became possible. Initially, such loans had short terms, so only simple interest was applied to the loan. However, when loans began to stretch out for years, it was natural to add the interest at the end of each year, and add the interest to the principal of the loan. After another year, the interest was calculated on the initial principal plus the interest from year 1, or, the interest earned interest. Each year, more interest was added to the money owed, and that interest continued to earn interest.

Since the amount in the account grows each year, more money earns interest, increasing the account faster. This growth follows a geometric series (Geometric Sequences). It is this feature that gives compound interest its power. This module covers the mathematics of compound interest.

Understand and Compute Compound Interest

As we saw in Simple Interest, an account that pays simple interest only pays based on the original principal and the term of the loan. Accounts offering **compound interest** pay interest at regular intervals. After each interval, the interest is added to the original principal. Later, interest is calculated on the original principal plus the interest that has been added previously.

After each period, the interest on the account is computed, then added to the account. Then, after the next period, when interest is computed, it is computed based on the original principal AND the interest that was added in the previous periods.