

6.1 Understanding Percent

THE FEDERAL BUDGET IN FISCAL YEAR 2021

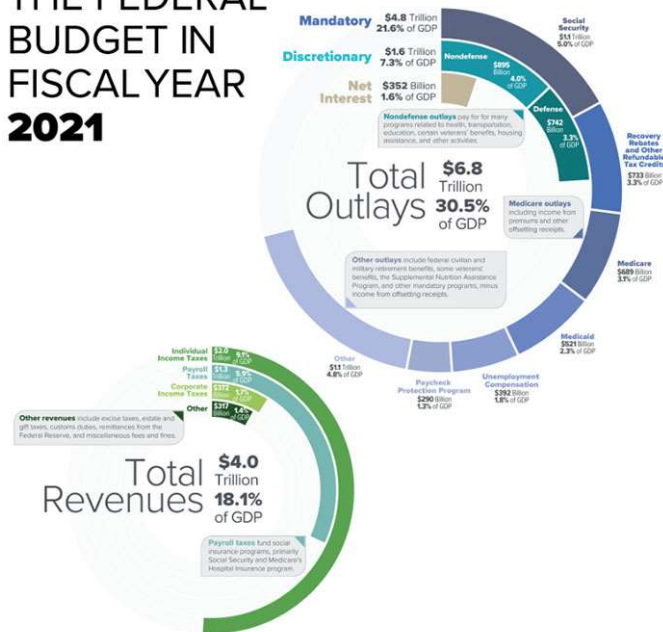


Figure 6.2 The federal budget describes how money is spent and how money is earned. (credit: "Breakdown of revenues and outlays in 2021 US Federal budget" Wikimedia Commons, Public Domain)

Learning Objectives

After completing this section, you should be able to:

1. Define and calculate percent.
2. Convert between percent, decimal, and fractional values.
3. Calculate the total, percent, or part.
4. Solve application problems involving percents.

In 2020, the U.S. federal government [budgeted \\$3.5 billion for the National Park Service](https://openstax.org/r/National_Park_Service) (https://openstax.org/r/National_Park_Service), which appears to be a very large number (and is!) and a large portion of the total federal budget. However, the [total outlays from the U.S. federal government in 2020 was \\$6.6 trillion](https://openstax.org/r/U.S._federal_government) (https://openstax.org/r/U.S._federal_government). So, the amount budgeted for the National Park Service was less than one-tenth of 1 percent, or 1/10%, of the total outlays. This **percent** describes a specific number. Understanding that ratio puts the \$3.5 billion budgeted to the National Park Service in perspective.

This chapter focuses on percent as a primary tool for understanding money management. The interest paid on debt, the interest earned through investments, and even taxes are entirely determined using percent. This section introduces the basics of working with this invaluable tool.

Define and Calculate Percent

The word **percent** comes from the Latin phrase per centum, which means “by the hundred.” So any percent is a number divided by 100. Changing a percent to a fraction is to write the percent in its **fractional form**. To write $n\%$ in its fractional form is to write the percent as the fraction $\frac{n}{100}$.

 A percent need not be an integer and does not have to be less than 100.

EXAMPLE 6.1

Rewriting a Percent as a Fraction

Rewrite the following as fractions:

1. 18%

2. 84%
3. 38.7%
4. 213%

✓ **Solution**

1. Using the definition and $n = 18$, 18% in fractional form is $\frac{18}{100}$.
2. Using the definition and $n = 84$, 84% in fractional form is $\frac{84}{100}$.
3. Using the definition and $n = 38.7$, 38.7% in fractional form is $\frac{38.7}{100}$.
4. Using the definition and $n = 213$, 213% in fractional form is $\frac{213}{100}$.

> **YOUR TURN 6.1**

Rewrite the following as fractions:

1. 3%
2. 94%
3. 67.2%
4. 670%

Convert Between Percent, Decimal, and Fractional Values

When any calculation with a percent is to be performed, the form of the percent must be changed, either to its fractional form or its **decimal form**. We can change a percent into decimal form by dividing the percent by 100 and representing the result as a decimal.

FORMULA

The decimal form of $n\%$ is found by calculating the decimal value of $n \div 100$.

EXAMPLE 6.2

Converting a Percent to Decimal Form

Convert the following percents to decimal form:

1. 17%
2. 7%
3. 18.45%

✓ **Solution**

1. To convert 17% to its decimal form, divide 17 by 100. This moves the decimal two places to the left, resulting in 0.17. The decimal form of 17% is 0.17.
2. To convert 7% to its decimal form, divide 7 by 100. This moves the decimal two places to the left, resulting in 0.07. The decimal form of 7% is 0.07.
3. To convert 18.45% to its decimal form, divide 18.45 by 100. This moves the decimal two places to the left, resulting in 0.1845. The decimal form of 18.45% is 0.1845.

> **YOUR TURN 6.2**

Convert the following percents to decimal form:

1. 9%
2. 24%
3. 2.18%

You should notice that, to convert from percent to decimal form, you can simply move the decimal two places to the left without performing the division.

FORMULA

To convert the number x from decimal form to percent, multiply x by 100 and place a percent sign, %, after the number, $(x \times 100)\%$.

EXAMPLE 6.3

Converting the Decimal Form of a Percent to Percent

Convert each of the following to percent:

1. 0.34
2. 4.15
3. 0.0391

Solution

1. Using the formula and $x = 0.34$, we calculate $(0.34 \times 100)\%$, which gives us 34%.
2. Using the formula and $x = 4.15$, we calculate $(4.15 \times 100)\%$, which gives us 415%.
3. Using the formula and $x = 0.0391$, we calculate $(0.0391 \times 100)\%$, which gives us 3.91%.

YOUR TURN 6.3

Convert the following to percent:

1. 0.41
2. 0.02
3. 9.2481

You should notice that, to convert from decimal form to percent form, you can simply move the decimal two places to the right without performing the multiplication.

Calculate the Total, Percent, or Part

The word “of” is used to indicate multiplication using fractions, as in “one-fourth of 56.” To find “one-fourth of 56” we would multiply 56 by one-fourth. We can think of percents as fractions with a specific denominator—100. So, to calculate “25% of 52,” we multiply 52 by 25%. But, first we need to convert the percent to either fractional form ($25/100$) or decimal form. Using the decimal form of 25% we have 0.25×52 , which equals 13.

In this problem, 52 is the **total** or **base**, 25 is the **percentage**, and 13 is **the percentage of 52**, or the **part** of 52. This is sometimes referred to as the **amount**.

FORMULA

The mathematical formula relating the total (base), the percent in decimal form, and the part (amount) is
 $part = percent \times total$, or, $amount = percent \times base$.

 In all calculations, the percent is expressed in decimal form.

Knowing any two of the values in our formula allows us to calculate the third value. In the following example, we know the total and the percent, and are asked to find the percentage of the total.

EXAMPLE 6.4**Finding the Percent of a Total**

1. Determine 70% of 3,500
2. Determine 156% of 720

✓ Solution

1. The total is $x = 3,500$, and the percent is $n = 70$. The decimal form of 70% is 0.70. To find the part, or percent of the total, substitute those values into the formula and calculate.

$$\begin{aligned}\text{part} &= \text{percent} \times \text{total} \\ &= 0.70 \times 3500 \\ &= 2450\end{aligned}$$

From this, we say that 70% of 3,500 is 2,450.

2. The total is $x = 720$, and the percent is $n = 156$. The decimal form of 156% is 1.56. To find the part, or percent of the total, substitute those values into the formula and calculate.

$$\begin{aligned}\text{part} &= \text{percent} \times \text{total} \\ &= 1.56 \times 720 \\ &= 1,123.2\end{aligned}$$

From this, we say that 156% of 720 is 1,123.2.

> YOUR TURN 6.4

1. Determine 26% of 1,300.
2. Determine 225% of 915.

▶ VIDEO

[Finding Percent of a Total \(https://openstax.org/r/solve_percent_problem1\)](https://openstax.org/r/solve_percent_problem1)

In the previous example, we knew the total and the percent and found the part using our formula. We may instead know the percent and the part, but not the total. We can use our formula again to solve for the total.

EXAMPLE 6.5**Finding the Total from the Percent and the Part**

1. What is the total if 35% of the total is 70?
2. What is the total if 10% of the total is 4,000?

✓ Solution

1. **Step 1:** The percent is 35, which in decimal form is 0.35. We were given that 35% of the total is 70, so the part is 70. We are to find the total. Substituting into the formula, we have

$$\begin{aligned}\text{part} &= \text{percent} \times \text{total} \\ 70 &= 0.35 \times \text{total}\end{aligned}$$

Step 2: To find the total, we solve the equation for the total.

$$\begin{aligned}70 &= 0.35 \times \text{total} \\ \frac{70}{0.35} &= \frac{0.35 \times \text{total}}{0.35} \\ 200 &= \frac{\cancel{0.35} \times \text{total}}{\cancel{0.35}} \\ 200 &= \text{total}\end{aligned}$$

From this we see that 200 is the total, or, that 35% of 200 is 70.

2. **Step 1:** The percent is 10, which in decimal form is 0.1. We were given that 10% of the total is 4,000, so the part is 4,000. Substituting into the formula, we have

$$\begin{aligned}\text{part} &= \text{percent} \times \text{total} \\ 4,000 &= 0.1 \times \text{total}\end{aligned}$$

Step 2: To find the total, we solve the equation for the total.

$$\begin{aligned}4,000 &= 0.1 \times \text{total} \\ \frac{4,000}{0.1} &= \frac{0.1 \times \text{total}}{0.1} \\ 40,000 &= \frac{\cancel{0.1} \times \text{total}}{\cancel{0.1}} \\ 40,000 &= \text{total}\end{aligned}$$

From this we see that 40,000 is the total, or that 10% of 40,000 is 4,000.

> YOUR TURN 6.5

1. What is the total if 18% of the total is 45?
2. What is the total if 15% of the total is 900?

▶ VIDEO

[Finding the Total from the Percent and the Part \(https://openstax.org/r/solve_percent_problem2\)](https://openstax.org/r/solve_percent_problem2)

Similarly, the percent can be found if the total and the percent of the total (the part) are known. This will result in the decimal form of the percent, so it must be converted to percent form.

EXAMPLE 6.6

Finding the Percent from the Total and the Part

1. What percent of 500 is 175?
2. What percent of 228 is 155?

✓ Solution

1. **Step 1:** The total is 500, the percent of the total is 175. Substituting into the formula, we have

$$\begin{aligned}\text{part} &= \text{percent} \times \text{total} \\ 175 &= \text{percent} \times 500\end{aligned}$$

Step 2: To find the percent, we solve the equation for the percent.

$$\begin{aligned}175 &= \text{percent} \times 500 \\ \frac{175}{500} &= \frac{\text{percent} \times 500}{500} \\ 0.35 &= \frac{\text{percent} \times \cancel{500}}{\cancel{500}} \\ 0.35 &= \text{percent}\end{aligned}$$

We see the percent in decimal form is 0.35. Converting from the decimal form yields 35%. We say that 175 is 35% of 500.

2. **Step 1:** The total is 228, the percent of the total is 155. Substituting into the formula, we have

$$\begin{aligned}\text{part} &= \text{percent} \times \text{total} \\ 155 &= \text{percent} \times 228\end{aligned}$$

Step 2: To find the percent, we solve the equation for the percent.

$$\begin{aligned} 155 &= \text{percent} \times 228 \\ \frac{155}{228} &= \frac{\text{percent} \times 228}{228} \\ 0.6798 &= \frac{\text{percent} \times \cancel{228}}{\cancel{228}} \\ 0.6798 &= \text{percent} \end{aligned}$$

We see the percent is 0.6798 (rounded to four decimal places). Converting from the decimal form yields 67.98%. We say that 155 is 67.98% of 228.

> YOUR TURN 6.6

Find the percent in the following:

1. Total is 40, percent of the total is 25
2. Total is 730, percent of the total is 292

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[Finding the Percent When the Total and the Part Are Known \(https://openstax.org/r/solve_percent_problem3\)](https://openstax.org/r/solve_percent_problem3)

Solve Application Problems Involving Percents

Percents are frequently used in finance, research, science experiments, and even casual conversation. Understanding these types of values helps when consuming media or discussing finances, for instance. Effectively working with and interpreting numbers and percents will help you become an informed consumer of this information.

In most cases, working through what is presented requires you to identify that you are indeed working with a question of percents, which two of the three values that are related through percents are known, and which of the three values you need to find.

EXAMPLE 6.7

Retention Rate at College

Justine applies to a medium size university outside her hometown and finds out that the retention rate (percent of students who return for their sophomore year) for the 2021 academic year at the university was 84%. During a visit to the registrar's office, she finds out that 1,350 people had enrolled in academic year 2021. How many students from the academic year 2021 are returning for the 2022 academic year?

✓ Solution

The percent of students who will return for the 2022 academic year (the retention rate) is 84%. The total number of students who enrolled in the 2021 academic year was 1,350. This means the percent is known and the total is known. From this, we can determine the number of students who will return (percent of the total) for the 2022 academic year using the formula $\text{part} = \text{percent} \times \text{total}$. Substituting into the formula and calculating, we find that the number of students that are returning is

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ &= 0.84 \times 1,350 \\ &= 1,134 \end{aligned}$$

So 1,134 students will return for the 2022 academic year.

> YOUR TURN 6.7

1. Harris works the bookstore in their hometown. During one particular day, the store had total sales of \$1,765, of

which Harris sold 30%. What were Harris's total sales that day?

EXAMPLE 6.8

Percent of Chemistry Majors

Cameron enrolls in a calculus class. In this class of 45 students, there are 18 chemistry majors. What percent of the class are chemistry majors?

✓ Solution

In this situation, the percent is to be determined. We know the total number of students, 45, and the part of the students that are chemistry majors, 18. Using that information and the formula $\text{part} = \text{percent} \times \text{total}$, the percent can be found. Substituting and solving, we have

$$\begin{aligned} 18 &= \text{percent} \times 45 \\ \frac{18}{45} &= \frac{\text{percent} \times 45}{45} \\ 0.4 &= \frac{\text{percent} \times \cancel{45}}{\cancel{45}} \\ 0.4 &= \text{percent} \end{aligned}$$

Converting the 0.4 from decimal form, we find that 40% of the students in the calculus class are chemistry majors.

> YOUR TURN 6.8

1. At the Fremont County fair, there were 2,532 adult visitors. Of these, 1,679 purchased the Adult Mega Pass. What percent of the adult visitors purchased the Adult Mega Pass?

EXAMPLE 6.9

Total Sales and Commission

Mariel makes a 20% commission on every sale she makes. One week, her commission check is for \$153.00. What were her total sales that week?

✓ Solution

In this problem, Mariel's total sales is to be determined. We know the percent she earns is 20%. We also know that her sales commission was \$153.00, which is the percent of the total. Using this information and the formula $\text{part} = \text{percent} \times \text{total}$ we can find Mariel's total sales. The decimal form of 20% is 0.2. The part, or percent of the total, is 153. Substituting and solving, we obtain

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ 153 &= 0.2 \times \text{total} \\ \frac{153}{0.2} &= \frac{0.2 \times \text{total}}{0.2} \\ 765 &= \frac{\cancel{0.2} \times \text{total}}{\cancel{0.2}} \\ 765 &= \text{total} \end{aligned}$$

Mariel's total sales were \$765.00.

> YOUR TURN 6.9

1. Mina's family has replaced 65% of their home's older light bulbs with LED bulbs. If they now have 52 LED bulbs, how many total lightbulbs are in Mina's house?

? WHO KNEW?**LED Lightbulbs**

According to the [energy website from the U.S. government \(https://openstax.org/r/U.S._government\)](https://openstax.org/r/U.S._government), LED lightbulbs use at least 75% less energy than incandescent bulbs. They also last up to 25 times as long as an incandescent bulb. If lighting is a significant percent of your electrical use, replacing all incandescent bulbs with LED bulbs will significantly reduce your electric bill.

Check Your Understanding

1. What is the denominator for any percent?
2. Convert 38.7% to decimal form.
3. What is 68% of 280?
4. Find the total if 41% of the total is 342. If necessary, round to two decimal places.
5. TikTok has an estimated 80,000,000 (80 million) registered users in the United States. The population of the United States is 332,403,650. What percent of the U.S. population are registered TikTok users? If necessary, round to two decimal places.
6. An Amazon fulfillment center needs to hire 20% more drivers. If there are currently 80 drivers, how many more drivers will be hired?

**SECTION 6.1 EXERCISES**

For any answer, round to two decimal places, if necessary.

In the following exercises, rewrite the percent as a fraction

1. 45%
2. 9.1%
3. 8%
4. 673%

In the following exercises, rewrite the percent in decimal form.

5. 18%
6. 9%
7. 71.2%
8. 934%
9. Find 35% of 250
10. Calculate 83.1% of 390
11. Calculate 3.1% of 500
12. Calculate 750% of 620
13. If 40% of the total is 32, how much is the total?
14. If 3% of the total is 6.32, how much is the total?
15. If 150% of the total is 61.9, how much is the total?
16. If 18.1% of the total is 18.5, how much is the total?
17. 13 is what percent of 40?
18. 89 is what percent of 500?
19. 31 is what percent of 73?
20. 593.2 is what percent of 184.5?
21. 36 people in a village of 150 want to install a new splashpad at the local playground. What percent of the village

wants to install the new splashpad?

22. Mitena is enrolled in a movie appreciation course. There are 84 students (including Mitena) in the course. After having the students fill out a survey, the professor informs the students that 45.2% chose horror as their favorite movie genre. How many students in Mitena's class chose horror as their favorite movie genre? Round off to the nearest integer.
23. Jady's dorm has a "Rick and Morty night" every Wednesday during the semester. One Wednesday, 27 students from the dorm come to watch the TV show Rick and Morty. Jady knows this is 30% of the dorm's residents. How many students reside in the dorm?
24. **Percent Error.** When performing a scientific experiment that results in quantities of some sort, such as mass in chemistry or momentum in physics, the percent error is often computed. Percent error, $%E$, is the percent by which the value obtained in an experiment, the observed value O , is different than the value that was expected, the expected value E , in the experiment. The formula is below.
- $$%E = \frac{|O-E|}{E}$$
- Jim and Kelly are working on a chemistry experiment and expect the result to be 50 grams. However, their result was 48.7 grams. Find Jim and Kelly's percent error.
25. **Percent Error.** See Exercise 24 for the definition of percent error. Hailey and Elsbeth are using an experiment to determine Earth's gravity. The expected value is 9.807 m/s^2 . Their experiment gives them a value of 9.457 m/s^2 . Find the percent error for Hailey and Elsbeth's experiment.

6.2 Discounts, Markups, and Sales Tax



Figure 6.3 Sale prices are often described as percent discounts. (credit: "Close-up of a discount sign" by Ivan Radic/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Calculate discounts.
2. Solve application problems involving discounts.
3. Calculate markups.
4. Solve application problems involving markups.
5. Compute sales tax.
6. Solve application problems involving sales tax.

Many people first encounter percentages during a retail transaction such as a percent discount (SALE! 25% off!), or through sales tax ("Wait, I thought this was \$1.99?"), a report that something has increased by some percentage of the previous value (NOW! 20% more!!). These are examples of percent decreases and percent increases. In this section, we