51. Does this pairwise election satisfy the IIA?

11.3 Standard Divisors, Standard Quotas, and the Apportionment Problem

Figure 11.15 Every person at a party gets their fair slice of the cake. (credit: "apple spice cake" by Mark Bonica/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Analyze the apportionment problem and applications to representation.
2. Evaluate applications of standard divisors.
3. Evaluate applications of standard quotas.

The Apportionment Problem

In the new democracy of Imaginaria, there are four states: Fictionville, Pretendstead, Illusionham, and Mythbury. Each state will have representatives in the Imaginarian Legislature. You might now have an agreement on which voting method your citizens will use to elect representatives. However, before that process can even begin, you must decided on how many representatives each state will receive. This decision will present its own challenges.

When sharing your birthday cake, it's only fair that everyone gets the same portion size, right? You were portioning the cake by dividing it up equally and giving everyone a slice. A great thing about cake is that you can slice it any way you want, but how do you **apportion**, or divide and distribute, items that can't be sliced? Suppose that you have a box of 16 Ring Pops™, gem-shaped lollipops on a plastic ring. You are going to share the box with four other kids. Dividing the 16 Ring Pops™ among the group of five leads to a problem; after each person in the group gets three Ring Pops™, there is still one left! Who gets the last one? The **apportionment problem** is how to fairly divide, or apportion, available resources that must be distributed to the recipients in whole, not fractional, parts.

The apportionment problem applies to many aspects of life, including the representatives in the Imaginarian legislature. The table below provides a short list of examples of resources that must be apportioned in whole parts, and the recipients of those resources.

Resource	Recipients
Covid-19 Vaccines	Nations around the world
Airport Terminals	Airlines
Faculty Positions at a University	Departments
Public Schools	Communities

Resource	Recipients
U.S. House of Representatives Seats	States
Parliamentary Seats	Political Parties

Fair division of a resource is not necessarily equal division of the resource like when distributing cake slices. When distributing airport terminals amongst airlines, there are many factors to consider such as the size of the airline, the number and types of aircraft they have, and the demand for the service. In most cases, fairness is defined as being **proportional**; two quantities are proportional if they have the same relative size. In the case of the Covid-19 vaccine, the expectation would be that countries with larger populations get more doses of the vaccine. In the Imaginarian legislature, the expectation may be that the states with larger populations will receive the larger number of representatives. This concept is referred to as a **part-to-part ratio**.

Suppose that a supermarket has a special on pies, two for $5. The first customer purchases four pies for $10, and the second customer purchases eight pies for $20. The dollar to pie ratio for the first customer is $\frac{10 \text{ dollars}}{4 \text{ pies}} = 2.5 \text{ dollars per pie}$ and the dollar to pie ratio for the second customer is $\frac{20 \text{ dollars}}{5 \text{ pies}} = 2.5 \text{ dollars per pie}$. So, the dollar to pie ratio is constant. Although the customers do not spend the same amount of money, the amount each spent was proportional to the number of pies purchased.

Now suppose that the supermarket changed the special to $5 for the first pie, and $2 for each additional pie. In that case, four pies would cost $\$5 + 3\,(\$2) = \$11$, while 8 pies would cost $\$5 + 7\,(\$2) = \$19$. The dollar to pie ratios would be $\frac{11 \text{ dollars}}{4 \text{ pies}} = 2.75 \text{ dollars per pie}$ and $\frac{19 \text{ dollars}}{8 \text{ pies}} = 2.375 \text{ dollars per pie}$, respectively. This special does not result in a constant part to part ratio. The dollars spent are not proportional to the number of pies purchased.

▶ **VIDEO**

What Is a Ratio? (https://openstax.org/r/ratios_proportions)

What Are the Different Types of Ratios? (https://openstax.org/r/ratio_types)

EXAMPLE 11.24

Ratio of Faculty to Students at a College

The following table provides a comparison of the number of faculty members in each department at a particular college to the student head count in that department and the number of class sections in that department in the Spring semester. Use this information to answer the questions.

Department	Mathematics	English	History	Science
(S) Student Head Count	4800	2376	1536	2880
(C) Class Sections	120	108	48	96
(T) Total Faculty	30	27	12	24
(F) Full-Time Faculty	10	9	4	8
(P) Part-Time Faculty	20	18	8	16

1. Determine the ratios for each department: S to C, C to T, S to T, F to P
2. What are the units of the ratios that you found?
3. Which of these pairs, if any, has a constant part to part ratio? State the ratio.
4. Does it appear that the total number of faculty positions were allocated to each department based on student head count, the number of class sections, or neither? Justify your answer.

Solution

1. Divide the first quantity by the second, for each department, as shown in the table below.
2. Answers are provided in last column of the table.

Department	eMathematics	English	History	Science	Units of Ratios Found
S to C	$\frac{4800}{120} = 40$	$\frac{2376}{108} = 22$	$\frac{1536}{48} = 32$	$\frac{2880}{96} = 30$	Students per class section
C to T	$\frac{120}{30} = 4$	$\frac{108}{27} = 4$	$\frac{48}{12} = 4$	$\frac{96}{24} = 4$	Class sections per faculty member
S to T	$\frac{4800}{30} = 160$	$\frac{2376}{27} = 88$	$\frac{1536}{12} = 128$	$\frac{2880}{24} = 120$	Students per faculty member
F to P	$\frac{10}{20} = \frac{1}{2}$	$\frac{9}{18} = \frac{1}{2}$	$\frac{4}{8} = \frac{1}{2}$	$\frac{8}{16} = \frac{1}{2}$	Full-time faculty member per part-time faculty member

3. The ratio of class sections to faculty members is a constant ratio of four. The ratio of full-time faculty to part-time faculty is a constant ratio of $\frac{1}{2}$.
4. It appears that the faculty positions were allocated based on the number of class sections because there is a constant ratio of four class sections per faculty member.

YOUR TURN 11.24

The SAT is to be administered at a high school. In preparation, pencils have been distributed to each of the classrooms based on the room capacity. Use the information in the following table to answer each question.

Room Number	B	C	D	E
Room Capacity (number of student desks)	24	18	32	22
Number of Pencils	36	27	48	33

1. Find the part to part ratio of desks to pencils for each room. Represent it as both a reduced fraction and a decimal rounded to the nearest hundredth as needed.
2. Find the part to part ratio of pencils to desks for each room. Represent it as both a reduced fraction and a decimal rounded to the nearest hundredth as needed.
3. Have the pencils been distributed proportionally? If so, what is the constant ratio of pencils to desks? Give the units.

There are some useful relationships between quantities that are proportional to each other. When there is a constant ratio between two quantities, the one quantity can be found by multiplying the other by that ratio. Remember the supermarket special on pies, 2 pies for $5? The ratio of dollars to pies is $\frac{5 \text{ dollars}}{2 \text{ pies}} = 2.5 \text{ dollars per pie}$ and the ratio of pies to dollars is $\frac{2 \text{ pies}}{5 \text{ dollars}} = 0.4 \text{ pies per dollar}$. These two values are reciprocals of each other, $\frac{1}{2.5} = 0.4$ and $\frac{1}{0.4} = 2.5$. This means that multiplying by one has the same effect as dividing by the other. This also means that knowing either constant ratio allows us to calculate the price given the number of pies. To find the cost of 20 pies, multiply by the ratio of dollars to pies or divide by the ratio of pies to dollars.

- $20 \text{ pies} \times 2.50 \text{ dollars per pie} = 50 \text{ dollars}$
- $20 \text{ pies} \div 0.4 \text{ pies per dollar} = 50 \text{ dollars}$

These patterns are true in general.

FORMULA

Let A be a particular item and B another such that there is a constant ratio of A to B

- ratio of B's to A's $= \frac{1}{\text{ratio of } A\text{'s to } B\text{'s}}$ and ratio of A's to B's $= \frac{1}{\text{ratio of } B\text{'s to } A\text{'s}}$
- units of A $=$ (units of B) $\times$ (ratio of A's to B's) $= \frac{\text{units of } B}{\text{ratio of } B\text{'s to } A\text{'s}}$
- units of B $=$ (units of A) $\times$ (ratio of B's to A's) $= \frac{\text{units of } A}{\text{ratio of } A\text{'s to } B\text{'s}}$

EXAMPLE 11.25

Ratio of Faculty to Students at a College

Refer to the information given in Example 11.24.

1. If there are 32 class sections each semester in the Fine Art department, and the same ratio is used to determine the number of faculty members, how many faculty members would you expect to see in the Fine Art department?
2. If the Health Sciences department has 6 full-time faculty members, how many part-time faculty members are in the department?

Solution

1. Multiply the number of class sections by the ratio of faculty members per class section to find the number of faculty. Since there are 4 faculty members per class, the number of faculty members in 32 classes should be $32.4 = 128$ faculty members.
2. Multiply the number of full-time faculty by the ratio of part-time to full-time to find the number of part-time. Since ratio of full-time faculty to part-time faculty at the college is $\frac{1}{2}$ or 1 full-time per 2 part time, the ratio of part-time to full-time is $\frac{2}{1} = 2$ part-time to 1 full-time; so the number of part-time faculty in a department with 6 full-time faculty should be $6.4 = 24$ part-time faculty.

YOUR TURN 11.25

Refer again to the table providing information on classroom capacities and pencil distribution.

Room Number	B	C	D	E
Room Capacity (number of student desks)	24	18	32	22
Number of Pencils	36	27	48	33

1. Determine the number of pencils that would be allocated to a classroom F with 28 desks by multiplying the number of desks by the ratio of pencils to desks.
2. Determine the number of pencils that would be allocated to a classroom F with 28 desks by dividing the number of desks by the ratio of desks to pencils.
3. Assuming pencils continued to be distributed in the same manner, if there were another classroom, G, that received 51 pencils, how many desks would we expect to find in the room?

The apportionment application that will be important to the founders of Imaginaria occurs in **representative democracies** in which elected persons represent a group. The United Kingdom, France, and India each have a parliament, and the United States has a Congress, just as Imaginaria will have a legislature! The citizens of a country must decide what portion of the representatives each group, such as a state or province or even a political party, will have. A larger portion of representatives means greater influence over policy.

EXAMPLE 11.26

Ratio of U.S. Representatives to State Population

Table 11.4 contains a list of the five U.S. states with the greatest number of representatives in the U.S. House of Representatives, along with the population of that state in 2021. Use the information in the table to answer the questions.

State	Representative Seats	State Population
(CA) California	53	39,613,000
(TX) Texas	36	29,730,300
(NY) New York	27	19,300,000
(FL) Florida	27	21,944,600
(PA) Pennsylvania	18	12,804,100

Table 11.4 The First through Fifth Ranked States by Number of Representatives (sources: https://www.census.gov/popclock/ [state population], https://www.britannica.com/topic/United-States-House-of-Representatives-Seats-by-State-1787120 [representative seats])

1. What is the ratio of State Population to Representative Seats for each state to the nearest hundred thousand?
2. What is the ratio of Representative Seats to State Population for each state rounded to seven decimal places?
3. What is the ratio of Representative Seats to State Population for each state rounded to six decimal places?
4. Does there appear to be a constant ratio? Justify your answer.

Solution

1. CA 700,000; TX 800,000; NY 700,000; FL 800,000; PA 700,000
2. CA 0.0000013; TX 0.0000012; NY 0.0000014; FL 0.0000012; PA 0.0000014
3. CA 0.000001; TX 0.000001; NY 0.000001; FL 0.000001; PA 0.000001
4. The ratio of State Population to Representative Seats seems to be either 700,000 or 800,000. There does appear to be a constant ratio of about 0.000001 of Representative Seats to State Population if we round off to the sixth decimal place.

YOUR TURN 11.26

Table 11.9 contains a list of the five U.S. states ranked sixth through tenth in the number of representatives in the U.S. House of Representatives, along with the population of that state in 2021. Use the information in the table to answer the questions.

State	Representative Seats	State Population
(IL) Illinois	18	12,804,100
(OH) Ohio	16	11,714,600

Table 11.5 The Sixth through Tenth Ranked U.S. States by Number of Representatives (sources: https://www.census.gov/popclock/ [state population], https://www.britannica.com/topic/United-States-House-of-Representatives-Seats-by-State-1787120 [representative seats])

State	Representative Seats	State Population
(MI) Michigan	14	9,992,430
(GA) Georgia	14	10,830,000
(NC) North Carolina	13	10,701,000

Table 11.5 The Sixth through Tenth Ranked U.S. States by Number of Representatives (sources: https://www.census.gov/popclock/ [state population], https://www.britannica.com/topic/United-States-House-of-Representatives-Seats-by-State-1787120 [representative seats])

1. What is the ratio of State Population to Representative Seats for each state to the nearest hundred thousand?
2. What is the ratio of Representative Seats to State Population for each state rounded to seven decimal places?
3. What is the ratio of Representative Seats to State Population for each state rounded to six decimal places?
4. Does there appear to be a constant ratio? Are the results similar to the top five states?

VIDEO

Math Antics – Rounding (https://openstax.org/r/rounding)

You might be wondering why the ratio doesn't appear to be quite the same depending on the rounding of the values. We will see that the key to this variation lies in the fractions. Just like the five children sharing 16 Ring Pops™, there are going to be leftovers and there are many methods for deciding what to do with those leftovers.

The Standard Divisor

There are two houses of congress in the United States: the Senate and the House of Representatives. Each state has two senators, but the number of representatives depends on the population of the state. The number of representative seats in the U.S. House of Representatives is currently set by law to be 435. In order to distribute the seats fairly to each state, the ratio of the population of the U.S. to the number of representative seats must be calculated. The ratio of the total population to the house size is called the **standard divisor**, and it is the number of members of the total population represented by one seat.

Although apportionment applies to many other scenarios, such as the pencil distribution during the SAT, the terminology of apportionment is based on the House of Representatives scenario. Thus, several government-related terms take on a more general meaning. The **states** are the recipients of the apportioned resource, the **seats** are the units of the resource being apportioned, the **house size** is the total number of seats to be apportioned, the **state population** is the measurement of the state's size, and the **total population** is the sum of the state populations.

FORMULA

$$\text{Standard Divisor} = \frac{\text{Total Population}}{\text{House Size}}$$

EXAMPLE 11.27

The Standard Divisor of the U.S. House of Representatives 2021

As of this writing, the Census.gov website U.S. Population clock showed a population of 332,693,997. There are 435 seats in the U.S. House of Representatives. Find the standard divisor rounded to the nearest tenth.

Solution

Dividing $330{,}147{,}881$ people by 435 seats, there are 758,960.6 people per representative.

YOUR TURN 11.27

1. By the end of the first U.S. Congress in 1791, there were 13 states, 65 representative seats, and approximately 3,929,214 citizens. Find the standard divisor rounded to the nearest tenth.

Whether the standard divisor is less than, equal to, or greater than 1 depends on the ratio of the population to the number of seats.

- The standard divisor will be equal to 1 if the total population is equal to the number of seats. This would mean that each member of the population is allocated their own personal seat.
- The standard divisor will be a number between 0 and 1 when the total population is less than the number of seats. This means that each member of the population is allocated more than one seat.
- The standard divisor will be a number greater than 1when the total population is greater than the number of seats. This means that a certain number of members of the population will share 1 seat.

If the population is five children and the house consists of five pieces of candy, the standard divisor is $\frac{5 \text{ children}}{5 \text{ candies}} = 1 \text{ child per candy}$ meaning each child gets one candy. If the population is five children and ten pieces of candy, the standard divisor is $\frac{5 \text{ children}}{10 \text{ candies}} = 0.5 \text{ child per candy}$ meaning that each child gets more than one candy. If the population is five children and four pieces of candy, the standard divisor is $\frac{5 \text{ children}}{4 \text{ candies}} = 1.25 \text{ child per candy}$ meaning that each child gets less than one candy.

If the seats in the Imaginarian legislature are distributed to the states based on population, then the house size will be less than the population and we should expect the standard divisor to be a number greater than 1.

EXAMPLE 11.28

School Resource Officers in Brevard County, Florida

The public schools in a certain county have been allotted 349 school resource officers to be distributed among 327 public schools attended by approximately 271,500 students.

1. Identify the states, seats, house size, state population, and total population in this apportionment scenario.
2. Describe the ratio the standard divisor represents in this scenario and calculate the standard divisor to the nearest tenth.

Solution

1. The states are the schools in that county. The seats are the school resource officers. The house size is the number of school resource officers, which is 349. The state population is the number of students in a particular school, which was not given. The total population consists of the sum of the school populations, which is 271,500.
2. The standard divisor is the ratio of the total population to the house size, which is the number of students served by each resource officer. Divide $271,500 \text{ students} \div 349 \text{ officers} = 777.9 \text{ students per officer}$.

YOUR TURN 11.28

The Hernandez family and the Higgins family went trick-or-treating together for Halloween last year. They returned with 313 pieces of candy, which they will now apportion to the families. The Hernandez family has three children and the Higgins family has four children.

1. Identify the states, seats, house size, state population, and total population in this apportionment scenario.
2. Describe the ratio the standard divisor represents in this scenario and calculate the standard divisor to four decimal places.

The Standard Quota

Once the standard divisor for the Imaginarian legislature is calculated, the next task is to determine the number of seats that each state should receive, which is referred to as the state's **standard quota**. Unless all the states have the same population, each state will receive a different number of seats because the quantities will be proportionate to the state populations. To determine those amounts, we will use an idea we learned earlier. Recall that, when the number of units

of item A is proportionate to the number of units of item B, we have: $\text{units of } A = \frac{\text{units of } B}{\text{ratio of } B\text{'s to } A\text{'s}}$

In this case, we are trying to calculate the number of seats a state should be apportioned, the state's standard quota. So A So A would refer to seats allocated to a particular state, while B would refer to the state population. This means that the ratio of B to A is the ratio of the total population to house size, which is the standard divisor. So in apportionment terms, we have the following formula.

FORMULA

$$\text{State's Standard Quota} = \frac{\text{State Population}}{\text{Standard Divisor}} \text{ seats}$$

EXAMPLE 11.29

The Standard Quota of the U.S. House of Representatives 2021

Example 27 outlined that the Census.gov website U.S. Population clock showed a population of 330,147,881, there are 435 seats in the U.S. House of Representatives, and the standard divisor was 758,960.6 people per representative. The state of California has a population of approximately 39,613,000. Use these values to determine the standard quota for California to two decimal places.

Solution

$$\text{California's standard Quota} = \frac{\text{California's population}}{\text{Standard Divisor}} = \frac{39{,}613{,}000}{758{,}960.6} = 52.19 \text{ representatives}$$

YOUR TURN 11.29

1. By the end of the first U.S. Congress in 1791, there were 13 states, 65 representative seats, and approximately 3,929,214 citizens. In that year, the state of Delaware had a population of approximately 59,000. people. Use this information and the standard divisor you found in Your Turn 11.27 to find Delaware's standard quota rounded to two decimal places.

EXAMPLE 11.30

Apportionment of Laptops in a Science Department

The science department of a high school has received a grant for 34 laptops. They plan to apportion them among their six classrooms based on each classroom's student capacity. Use the values in the table below to find the standard quota for each classroom.

Room	Students
A	30
B	25
C	28
D	32
E	24
F	27

Solution

Step 1: Identify the state population, total population, and the house size. The states are the classrooms, and the state populations are listed in the table. The total population is the sum of the state populations, which is 166. The house size is the number of seats, or laptops, to be allocated, which is 34.

Step 2: Calculate the standard divisor by dividing the total population by the house size.

$$\text{Standard Divisor} = \frac{\text{Total of Room Capacities}}{\text{Number of Laptops}} = \frac{166}{34} \approx 4.88 \text{students per laptop.}$$

Step 3: Calculate the standard quota by dividing the state population by the standard divisors , as shown in the table below. $\text{Room's Standard Quota} = \frac{\text{Room Capacity}}{\text{Standard Divisor}}$

Room	Room Capacity	Room's Standard Quota
A	30	$30 \div 4.88 \approx 6.15$ laptops
B	25	$25 \div 4.88 \approx 5.12$ laptops
C	28	$28 \div 4.88 \approx 5.74$ laptops
D	32	$32 \div 4.88 \approx 6.56$ laptops
E	24	$24 \div 4.88 \approx 4.92$ laptops
F	27	$27 \div 4.88 \approx 5.53$ laptops

Step 4: Find the sum of the standard quotas. $6.15 + 5.12 + 5.74 + 6.56 + 4.91 + 5.53 = 34.01$. This is only slightly off from the number of laptops—34—which can be caused by rounding off in previous steps. This is a good indication that the calculations were correct. If you find that the value of the sum of the standard quotas is significantly different from the house size (number of seats), it is possible that the standard divisor was calculated using too few decimal places. Calculate the standard divisor and standard quotas again but round off to a greater number of decimal places.

YOUR TURN 11.30

1. This year the Hernandez family and the Higgins family were joined by the Ho family for Halloween trick-or-treating. The Hernandez family has three children, the Higgins family has four children, and the Ho family has two children. This time, they collected 527 pieces of candy, which they are going to apportion based on the number of children. Find the standard quota for each family. Round all values to four decimal places. If traditional rounding methods are applied to determine the actual number whole number values of pieces of candy received by each family, do the values sum to 527?

Check Your Understanding

In the following questions, assume there is a constant ratio between units of A and units of B. Two students are having a discussion. Determine who is correct: Student 1, Student 2, both, or neither.

18. Student 1 says that the number of units of A is the product of the number of units of B times the ratio of A to B.
    Student 2 says that the number of units of A is the quotient of the number of units of B divided by the ratio of A to B.
19. Student 1 says that the number of units of A is the quotient of the number of units of B divided by the ratio of B to A.
    Student 2 says that the number of units of B is the quotient of the number of units of A divided by the ratio of A to B.
20. Suppose there are 110 of item A and 55 of item B.

Student 1 says the ratio of A to B is $\frac{1}{2}$ Student 2 says the ratio of B to A is $\frac{1}{0.5}$.

21. Suppose the constant ratio of A to B is 0.75. Student 1 says the ratio of A to B is $\frac{4}{3}$ Student 2 says the ratio of B to A is $\frac{3}{4}$.
22. Student 1 says that $\text{Standard Divisor} = \frac{\text{House Size}}{\text{Total Population}}$. Student 2 says that $\text{Standard Divisor} = \frac{\text{Sum of the State Populations}}{\text{Total Number of Seats Available}}$.
23. Student 1 says that $\text{State's Standard Divisor} = \frac{\text{Total Population}}{\text{State's Standard Quota}}$. Student 2 says that $\text{State's Standard Quota} = \frac{\text{State Population}}{\text{Standard Divisor}}$.
24. Student 1 says that $\text{State's Standard Quota} = \text{State Population} \div \frac{\text{Total Population}}{\text{House Size}}$. Student 2 says that $\text{State's Standard Quota} = \frac{\text{State Population}}{\text{Standard Divisor}}$

SECTION 11.3 EXERCISES

For the following exercises, identify the states, the seats, and the state population (the basis for the apportionment) in the given scenarios.

1. A parent has 25 pieces of candy to split among their four children. They will earn the candy based on how many minutes of chores they children did this week.
2. The board of trustees of a college has recently approved the installation of 70 new emergency blue lights in five parking lots. The number of lights in each lot will be proportionate to the size of the parking lot, which is to be measured in acres.
3. The reading coach at an elementary school has 52 prizes to distribute to students as a reward for time spent reading.
4. "Top officials from Operation Warp Speed, the [U.S.] government's program to fast-track the development and delivery of COVID-19 vaccines, announced they've allocated 6.4 million doses of COVID-19 vaccines to states based on their total populations." *(The Coronavirus Crisis*, by Pien Huang, Shots Health News From NPR, npr.org, November 24, 2020)
5. Refer to question 4, except suppose that the COVID-19 vaccine allocations were based on the most vulnerable population, residents aged 65 and over.

For the following exercises, use the given information to find the standard divisor to the nearest hundredth. Include the units.

6. The total population is 2,235 automobiles, and the number of seats is 14 warehouses.
7. The total population is 135 hospitals, and the number of seats is 200 respirators.

For the following exercises, use the given information to find the standard quota. Include the units.

8. The state population is eight residents in a unit, and the standard divisor is 1.75 residents per parking space.
9. The state population is 52 ICU patients each week, and the number of seats is 6.5 patients per respirator.
10. The total population is 145 basketball players, the number of seats is 62 trophies, and the state population is 14 basketball players on Team Tigers.
11. The total population is 12 giraffes, the number of seats is nine water troughs, and the state population is three giraffes in Enclosure C.

For the following exercises, use the table below, which shows student head count, class section, and total faculty in each of four college departments.

Department	(M) Math	(E) English	(H) History	(S) Science	(O) College Overall
(T) Student Head Count	4800	2376	1536	2880	87118
(C) Class Sections	120	108	48	96	3712
(F) Faculty Members	30	27	12	24	928

12. Determine the F to T ratios for each department rounded to four decimal places as needed. What are the units?
13. Determine the C to F ratios for each department rounded to four decimal places as needed. What are the units?
14. What is the F to T ratio for the college overall? Include units. How does it compare to the F to T ratios for

individual departments?

15. What is the overall C to F ratio? Include units. How does it compare to the C to F ratios for individual departments?
16. Does there appear to be a constant F to T ratio? If so, what is the ratio? If not, what implications does this have about the different departments?
17. Does there appear to be a constant C to F ratio? If so, what is the ratio? If not, what implications does this have about the different departments?
18. If the departments are the states, the students are the population, and the faculty members are the seats, use the College Overall column to determine the standard divisor for the apportionment of the faculty rounded to two decimal places as needed. Include the units.
19. If the departments are the states, the classes are the population, and the faculty members are the seats, use the Overall College column to determine the standard divisor rounded to two decimal places as needed. Include the units.
20. Use the standard divisor from question 18 to find the standard quota for each department rounded to two decimal places as needed. What are the units?
21. Use the standard divisor from question 19 to find the standard quota for each department rounded to two decimal places as needed.

For the following exercises, use this information: Wakanda, the domain of the Black Panther, King T'Challa has six fortress cities. In Wakandan, the word "birnin" means "fortress city." King T'Challa has found 111 Vibranium artifacts that must be distributed among the fortress cities of Wakanda. He has decided to apportion the artifacts based on the number of residents of each birnin. The table below displays the populations of major Wakandan cities.

Fortress Cities	Birnin Djata (D)	Birnin T'Chaka (T)	Birnin Zana (Z)	Birnin S'Yan (S)	Birnin Bashenga (B)	Birnin Azzaria (A)	Total Population
Residents	26,000	57,000	27,000	18,000	64,000	45,000	237,000

22. Identify the states, the seats, and the state population (the basis for the apportionment) in this scenario.
23. Find the standard divisor for the apportionment of the Vibranium artifacts. Round to the nearest tenth as needed. Include the units.
24. Find each birnin's standard quota for the apportionment of the Vibranium artifacts. Round to the nearest hundredth as needed. What are the units?
25. Find the sum of the standard quotas. Is it reasonably close to the number of artifacts available for distribution?

For the following exercises, suppose that 6.4 million doses of COVID-19 vaccine are to be distributed among U.S. states. The vaccines will either be distributed based on the total state population or based on the number of people over 65 years old, as shown in the table below.

State	State Population	State Population Age 65+	Percentage of State Population 65+
(CA) California	39,613,000	5,669,000	14.3%
(TX) Texas	29,730,300	3,602,000	12.6%
(NY) New York	19,300,000	3,214,000	16.4%
(FL) Florida	21,944,600	4,358,000	20.5%
(PA) Pennsylvania	12,804,100	2,336,000	18.2%
(US) United States	330,151,000	52,345,000	15.8%

26. Find the standard divisor for the apportionment of the vaccine doses by population using the estimate for the total U.S. population. Round to the nearest tenth as needed. Include the units.
27. Find each state's standard quota for the apportionment of the vaccine doses. Round to the nearest tenth as

needed. What are the units?

28. Find the standard divisor for the apportionment of the vaccine doses by population age 65 and older using the estimate for the total U.S. population of people aged 65 and older. Round to the nearest tenth as needed. Include the units.
29. Find each state's standard quota for the apportionment of the vaccine doses by total state population. Round to the nearest tenth as needed. What are the units?
30. Compare the standard quota for each state based on the entire state population to the standard quota for each state based on the portion of the population age 65 and older. Which states would receive more doses of vaccine if the apportionment were based on the population of people age 65 and older?
31. Approximately 15.8 percent of U.S. residents are age 65 and older.
    a. Which of the five states listed have a percentage of residents age 65 and older greater than 15.8 percent?
    b. Which of the five states listed have a percentage of residents age 65 and older less than 15.8 percent?
    c. Explain the correlation.

For the following exercises, use this information: Children from five families—the Chorro family, the Eswaran family, the Javernick family, the Lahde family, and the Stolly family—joined a town-wide Easter egg hunt. When they returned with their baskets, they had 827 eggs! They decided to share their eggs among the families based on the number of children in each family, as shown in the table below.

Family	Number of Children
(C) Chorro	3
(E) Eswaran	2
(J) Javernick	4
(L) Lahde	1
(S) Stolly	5

32. Identify the states, the seats, and the state population (the basis for the apportionment) in this scenario.
33. Find the standard divisor for the apportionment of the Easter eggs. Round to five decimal places as needed. Include the units.
34. Find each family's standard quota for the apportionment of the Easter eggs. Round to the nearest hundredth as needed. What are the units?
35. Find the sum of the standard quotas from exercise 34. Is the sum reasonably close to the number of Easter eggs available for distribution?